**Power Spectrum**

1. **Power Spectrum**

The power spectrum answers the question “How much of the signal is at a frequency ”. The harmonic signals give peaks at a fundamental, quasiperiodic signals give peaks at linear combination of two or more irrotationally related frequencies. The Fourier transform of a deterministic signal is given by

The power spectrum is then given by , which follows from Parseval’s theorem that the signal energy is given by

* **Finite Stochastic Process**

Stochastic process does not have a Fourier transform, Fourier transform can only be applied to deterministic functions of time, but not random variables whose individual realization are such functions. In order to obtain the spectral distribution of power versus frequency for stochastic process, it is best to avoid infinite interval to begin with and start with a finite interval .

Formally, partial Fourier transform of a process based on is given by

The power distribution associated with the realization based on is given by

The function above represents a random variable for every and its ensemble gives the average power distribution based on . Therefore, the expected power is

For weak-sense-stationary (w.s.s) process[[1]](#footnote-1), it is possible to further simplify the equation. Thus if is assumed to be w.s.s then , letting , the equation simplifies to

* **Infinite Stochastic Process**

Finally, letting we obtain the power spectral density of the w.s.s process

We notice that , that is the autocorrelation function and the power spectrum of a w.s.s process forms a Fourier transform pair, a relation known as the **Wiener-Khinchin Theorem**. The inverse formula is

In particular, for we get the total power

1. A random process is called weak-sense-stationary if its mean function and its correlation function do not change by shifts in time. For a continuous random process to be a weak-sense stationary

   * Mean: for all
   * CF: for all

   [↑](#footnote-ref-1)